Generalized Weingarten Surfaces of the Radial Support Type

Laredo Rennan Pereira Santos (laredorennan@hotmail.com)
IFG

Abstract. I intend to do a poster submission about surfaces that can be parametrized as envelopes of a sphere congruence whose other envelope is contained in a unit sphere. Such surfaces has a local parametrization given by

$$X(u) = Y(u) - 2\left(\frac{h(u)+c}{S(u)}\right)\eta(u) \quad u \in U, \tag{1}$$

where $h : U \subset \mathbb{R}^n \to \mathbb{R}$ is a real differentiable function associated with the parametrization $Y : U \subset \mathbb{R}^n \to \mathbb{S}^n$ of $\mathbb{S}^n$ and

$$\eta = \nabla_L h + hY, \qquad S = \langle \eta, \eta \rangle = \left|\nabla_L h\right|^2 + h^2,$$

with $L_{ij} = \langle Y_{,i}, Y_{,j} \rangle$.

In my work it is exhibited a suficient condition to exist such sphere congruence. Moreover its radius function is given explicitly and it is proved to be a geometric invariant of the surface.

The characterization of surfaces that are associated to $\mathbb{S}^n$ by a sphere congruence is used to study a class of generalized Weingarten surfaces, named generalized Weingarten surfaces of the radial support type - RSGW- surfaces in short - which satisfy a differentiable relation between the mean and Gaussian curvatures, the support function and the radius function from the sphere congruence.

Under certain condition, a surface locally parametrized as in (1) is a RSGW-surface if, and only if, the function h is harmonic. In this case, the vector function η is a local parametrization for a Appell surface. Futhermore, there is a Weierstrass type representation depending on two holomorphic functions for the RSGW-surfaces and, consequently, for the Appell surfaces locally parametrized by η.

References

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